τ mesonic decays and strong anomaly of PCAC

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Abstract

It is shown that the matrix elements of the quark axial-vector currents of the decays $\tau \rightarrow \omega\pi\pi\nu$ and $\omega\rho\nu$ are not conserved in the limit of $m_q = 0$. Pion exchange dominates the decay mode $\tau \rightarrow \omega\rho\nu$. Theoretical result of $\tau \rightarrow \omega(\pi\pi)_{non\rho}\nu$ agrees with data well. Both the decay modes provide evidences for the existence of strong anomaly of the PCAC. The strong anomaly originates in the Wess-Zumino-Witten anomaly. The PCAC with strong anomaly is written down and is applied to study $\omega \rightarrow \pi\gamma$, $\rho \rightarrow \pi\gamma$, and $\omega \rightarrow 3\pi$ under the soft pion approximation. Theoretical results are in good agreement with data. The decay $\tau \rightarrow K^*\rho\nu$ and $K^*\omega\nu$ and the PCAC ($\Delta s = 1$) with strong anomaly is presented.

The hadrons in τ hadronic decays are mesons, therefore the τ mesonic decays provide a test ground for all meson theories, especially the anomaly in meson physics. $\pi^0 \rightarrow 2\gamma$ is via the PCAC related to the Adler-Bell-Jackiw triangle anomaly[1]. The Adler-Bardeen theorem[2] is about the anomaly in QCD. The Wess-Zumino-Witten(WZW)[3,4] Lagrangian provides a general formalism for various abnormal meson processes. In Ref.[5] an effective chiral theory of mesons has been proposed. In this theory the WZW Lagrangian is the leading term of the imaginary part of the Lagrangian of the effective chiral theory. The fields in the WZW Lagrangian are normalized to the physical meson fields. Based on chiral symmetry and chiral symmetry breaking, a bosonized axial-vector currents of ordinary quarks has been presented in our recent paper[6]. In this paper the vector currents of ordinary quarks are treated by the VMD and all the meson vertices are obtained from the effective chiral theory[5]. The τ mesonic decays are studied in terms of this theory[6]. Some of the decay modes are related to the WZW anomaly. Theoretical results are in reasonable agreement with data. In the τ-decay modes studied in [6] the a_1 meson is dominant in the matrix elements of the axial-vector currents(u and d quarks) and the axial-vector currents are conserved in the chiral limit.

In this paper the decays $\tau \rightarrow \omega(\pi\pi)_{non\rho}\nu$ and $\tau \rightarrow \omega\rho\nu$ are studied. Only axial-vector currents contribute to these decays. It is found that the matrix elements of the axial-vector currents are not conserved in the chiral limit and pion dominates the decay $\tau \rightarrow \omega\rho\nu$. These

abnormal phenomena originate in the anomaly of PCAC. Here the anomaly is the one of strong interaction. The strong interaction anomaly of PCAC is studied in this paper. The study on the decays $\tau \to K^*\rho\nu$ and $K^*\omega\nu$ is presented. It is found in these two decays that the axial-vector currents($\Delta s = 1$) is not conserved in the limit of $m_q = 0$. The strong anomaly of the PCAC($\Delta s = 1$) is investigated.

In the chiral limit, the vector part of weak interaction of ordinary quarks(u and d quarks only) is written as[6]

$$\mathcal{L}^V = \frac{g_W}{4}cos\theta_C\frac{1}{f_\rho}\{-\frac{1}{2}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial_\mu\rho^i_\nu - \partial_\nu\rho^i_\mu) + A^i_\mu j^{i\mu}\}, \tag{1}$$

where $i = 1, 2$ and A^i_μ are W boson fields, j^i_μ is derived by using the substitution

$$\rho^i_\mu \to \frac{g_W}{4f_\rho}cos\theta_C A^i_\mu \tag{2}$$

in the vertices involving ρ mesons. In terms of the chiral symmetry and spontaneous chiral symmetry breaking, the axial-vector part has been determined as

$$\begin{aligned}\mathcal{L}^A = &-\frac{g_W}{4}cos\theta_C\frac{1}{f_a}\{-\frac{1}{2}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial_\mu a^i_\nu - \partial_\nu a^i_\mu) + A^{i\mu}j^{iW}_\mu\}\\ &-\frac{g_W}{4}cos\theta_C\Delta m^2 f_a A^i_\mu a^{i\mu} - \frac{g_W}{4}cos\theta_C f_\pi A^i_\mu\partial^\mu\pi^i,\end{aligned} \tag{3}$$

where f_a and Δm^2 are determined to be

$$f^2_a = f^2_\rho(1 - \frac{f^2_\pi f^2_\rho}{m^2_\rho})\frac{m^2_a}{m^2_\rho}, \quad \Delta m^2 = f^2_\pi(1 - \frac{f^2_\pi f^2_\rho}{m^2_\rho})^{-1}, \tag{4}$$

$j_\mu^{i,W}$ is obtained by substituting

$$a_\mu^i \to -\frac{g_W}{4f_a} cos\theta_C A_\mu^i \tag{5}$$

into the Lagrangian in which a_1 meson is involved.

The Lagrangians $\mathcal{L}^{V,A}$(1,3) have been derived from the effective chiral theory of pseudoscalar, vector, and axial-vector mesons[5]. The Lagrangian of this theory is expressed as

$$\begin{aligned}\mathcal{L} = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi}M\psi \\ +\frac{1}{2}m_0^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu)\end{aligned} \tag{6}$$

where $a_\mu = \tau_i a_\mu^i + f_\mu$, $v_\mu = \tau_i\rho_\mu^i + \omega_\mu$, , and $u = exp\{i\gamma_5(\tau_i\pi_i + \eta)\}$, these fields are normalized to physical meson fields in Ref.[5]. The parameters of Eqs.(1,3) are defined as[5]

$$f_\rho = g^{-1}, \tag{7}$$

$$f_a = g^{-1}(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}, \tag{8}$$

$$(1 - \frac{1}{2\pi^2 g^2})m_a^2 = 6m^2 + m_\rho^2, \tag{9}$$

$$\Delta m^2 = 6m^2 g^2 = f_\pi^2(1 - \frac{f_\pi^2}{g^2 m_\rho^2})^{-1}, \tag{10}$$

where g is an universal coupling constant, $g = 0.39$[6], and m is a parameter related to quark condensate[5].

It has been shown in Ref.[6] that there are cancellations between the terms of Eq.(3) in the decay modes studied and these cancellations lead to both the conservation of the axial-vector currents in the chiral limit and the a_1 dominance.

The meson vertices involved in τ mesonic decays are obtained from the effective Lagrangian of mesons presented in Ref.[5]. There are two kinds of vertices: the ones of normal parity and the ones of abnormal parity. The later are the WZW anomaly. Therefore, the theory of τ mesonic decays is completely determined by the effective chiral theory of mesons[5].

The ω meson is contained in the final states of both the decay modes $\tau \to \omega\pi\pi\nu$ and $\tau \to \omega\rho\nu$. It is well known[4,5,7] that in two flavor case if a vertex contains ω-field, the vertex is from WZW anomaly. Therefore, both decay modes are related to the WZW anomaly.

The decay mode $\tau \to \omega\pi\pi\nu$ is composed of two parts: the two pions are from a ρ decay and the two pions are not from a ρ resonance. The study(see below) shows that the decay rate of $\tau \to \omega(\pi\pi)_{\rho res.}\nu$ is smaller than that of $\tau \to \omega(\pi\pi)_{non\rho}\nu$ by two order of magnitude. We study $\tau \to \omega(\pi\pi)_{non\rho}\nu$ first.

Only the axial-vector currents($\mathcal{L}^A(3)$) contribute to $\tau \to \omega(\pi\pi)_{non\rho}\nu$. There are two kinds of vertices involved in this decay channel. These vertices are derived from the effective chiral theory of mesons[5] in the chiral limit.

1. The vertices $\omega\pi\pi\pi$ and $\omega a_1\pi\pi$ derived from Ref.[5] are

$$\mathcal{L}^{\omega\pi\pi\pi} = \frac{2}{\pi^2 g f_\pi^3}(1-\frac{6c}{g}+\frac{6c^2}{g^2})\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\omega_\mu\partial_\nu\pi_i\partial_\alpha\pi_j\partial_\beta\pi_k$$
$$\mathcal{L}^{\omega a_1\pi\pi} = -\frac{6}{\pi^2 g^2 f_\pi^2}(1-\frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}(1-\frac{2c}{g})\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\partial_\nu\omega_\mu a^i_\alpha\partial_\beta\pi_k, \tag{11}$$

where

$$c = \frac{f_\pi^2}{2gm_\rho^2}. \tag{12}$$

These two vertices are from the WZW anomaly[5]. The vertex $\mathcal{L}^{W\omega\pi\pi}$ is found by using the substitution(5) in the vertex $\mathcal{L}^{\omega a_1\pi\pi}$. Using these vertices and $\mathcal{L}^A$(3), the matrix element of the axial-vector current is obtained

$$<\omega\pi^0\pi^-|\bar\psi\tau_+\gamma_\mu\gamma_5\psi|0>^{(1)} = \frac{i}{\sqrt{8\omega_1\omega_2 E}}\frac{6}{\pi^2 g f_\pi^2}\varepsilon^{\nu\lambda\alpha\beta}\epsilon^*_\lambda p_\alpha(k_2-k_1)_\beta$$
$$\{(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu})(1-\frac{2c}{g})\frac{g^2 f_a^2 m_\rho^2 - i\sqrt{q^2}\Gamma_a(q^2)}{q^2-m_a^2+i\sqrt{q^2}\Gamma_a(q^2)}+\frac{q_\mu q_\nu}{q^2}\frac{2c}{g}(1-\frac{2c}{g})\}, \tag{13}$$

where k_1, k_2, and p are momentum of π^0, π^-, and ω respectively, $q = k_1+k_2+p$. Eq.(13) shows that the matrix element obtained from the WZW anomaly is not conserved in the limit of $m_q = 0$. The pion exchange is dominant in the term which violates the conservation of the quark axial-vector current in the chiral limit. The divergence of this term is written as

$$\frac{6}{\pi^2 g f_\pi^2}\frac{2c}{g}(1-\frac{2c}{g})\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\partial_\nu\omega_\mu\partial_\alpha\pi^j\partial_\beta\pi^k. \tag{14}$$

2. The second kind of vertices are vertices $a_1\rho\pi$, $\rho\pi\pi$, $W\rho\pi$ and $\omega\rho\pi$. The vertex $W\rho\pi$ is derived by substituting(5) into the vertex $a_1\rho\pi$. The first three vertices have been exploited to study $\tau \to \pi\pi\pi\nu$ and theoretical results are in reasonably agreement with data. The vertex $\omega\rho\pi$ is from the WZW anomaly.

$$\mathcal{L}^{a_1\rho\pi} = \epsilon_{ijk}\{Aa^i_\mu\rho^{j\mu}\pi^k - Ba^i_\mu\rho^j_\nu\partial^{\mu\nu}\pi^k + Da^i_\mu\partial^\mu(\rho^j_\nu\partial^\nu\pi^k)\} \tag{15}$$

$$A = \frac{2}{f_\pi}gf_a\{g^2f_a^2m_a^2 - m_\rho^2 + p^2[\frac{2c}{g} + \frac{3}{4\pi^2g^2}(1-\frac{2c}{g})]$$
$$+q^2[\frac{1}{2\pi^2g^2} - \frac{2c}{g} - \frac{3}{4\pi^2g^2}(1-\frac{2c}{g})]\}, \tag{16}$$

$$B = -\frac{2}{f_\pi}gf_a\frac{1}{2\pi^2g^2}(1-\frac{2c}{g}), \tag{17}$$

$$D = -\frac{2}{f_\pi}f_a\{2c + \frac{3}{2\pi^2g}(1-\frac{2c}{g})\}, \tag{18}$$

$$\mathcal{L}^{\rho\pi\pi} = \frac{2}{g}\epsilon_{ijk}\rho^i_\mu\pi^j\partial^\mu\pi^k - \frac{2}{\pi^2f_\pi^2g}\{(1-\frac{2c}{g})^2 - 4\pi^2c^2\}\epsilon_{ijk}\rho^i_\mu\partial_\nu\pi^j\partial^{\mu\nu}\pi^k$$
$$-\frac{1}{\pi^2f_\pi^2g}\{3(1-\frac{2c}{g})^2 + 1 - \frac{2c}{g} - 8\pi^2c^2\}\epsilon_{ijk}\rho^i_\mu\pi_j\partial^2\partial_\mu\pi_k, \tag{19}$$

$$\mathcal{L}^{\omega\rho\pi} = -\frac{N_C}{\pi^2g^2f_\pi}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\rho^i_\alpha\partial_\beta\pi^i, \tag{20}$$

where p is the momentum of ρ meson and q is the momentum of a_1.

Using the vertex $\mathcal{L}^{a_1\rho\pi}$(15), the decay width of a_1 meson is derived

$$\Gamma_a(q^2) = \frac{k}{12\pi m_a\sqrt{q^2}}\{(3+\frac{k^2}{m_\rho^2})A^2 - \frac{k^2}{m_\rho^2}(q^2+m_\rho^2)AB + \frac{q^2}{m_\rho^2}k^4B^2\}, \tag{21}$$

where

$$k = \{\frac{1}{4q^2}(q^2+m_\rho^2-m_\pi^2)^2 - m_\rho^2\}^{\frac{1}{2}}.$$

Using $\mathcal{L}^A(3)$ and the vertices(15,19,20) and taking the cancellation shown in Eq.(34) of Ref.[6] into account, the second part of the matrix element of the axial-vector current is obtained

$$
\begin{aligned}
&< \omega\pi^0\pi^-|\bar{\psi}\tau_+\gamma_\mu\gamma_5\psi|0>^{(2)}= \frac{i}{\sqrt{8\omega_1\omega_2 E}}(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})\frac{3}{\pi^2 g^2 f_\pi}\frac{g^2 f_a m_\rho^2 - i f_a^{-1} q\Gamma_a(q^2)}{q^2 - m_a^2 + iq\Gamma_a(q^2)} \\
&\varepsilon^{\nu\lambda\alpha\beta}\epsilon^*_\lambda p_\alpha\{\frac{A(k^2)k_{2\beta}}{k^2 - m_\rho^2 + i\sqrt{k^2}\Gamma_\rho(k^2)} - \frac{A(k^{'2})k_{1\beta}}{k'^2 - m_\rho^2 + i\sqrt{k'^2}\Gamma_\rho(k'^2)}\} \\
&+\frac{i}{\sqrt{8\omega_1\omega_2 E}}(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})\frac{3}{\pi^2 g^2 f_\pi}\frac{g^2 f_a m_\rho^2 - iq f_a^{-1}\Gamma_a(q^2)}{q^2 - m_a^2 + iq\Gamma_a(q^2)}\varepsilon^{\sigma\lambda\alpha\beta}\epsilon^*_\sigma p_\lambda k_{2\alpha}k_{1\beta}(-B) \\
&\{\frac{k_{2\nu}}{k'^2 - m_\rho^2 + i\sqrt{k'^2}\Gamma_\rho(k'^2)} + \frac{k_{1\nu}}{k^2 - m_\rho^2 + i\sqrt{k^2}\Gamma_\rho(k^2)}\},
\end{aligned} \tag{22}
$$

where k_1, k_2, p are momentum of π^0, π^-, and ω mesons respectively, $q = p + k_1 + k_2$, $k = q - k_1$, $k^{'} = q - k_2$, $A(k^2)$ and $A(k^{'2})$ are defined by Eq.(16) by taking $p^2 = k^2, k^{'2}$ respectively. This part of the matrix element observes the axial-vector current conservation in the limit $m_q = 0$ and there is a_1 dominance.

Adding Eqs.(13,22) together, the whole matrix element is obtained. The expression of the decay width is presented in the Appendix. The branching ratio is computed to be

$$B = 0.37\%.$$

The data is $0.41 \pm 0.08 \pm 0.06\%$[7]. The distribution of the decay rate versus the invariant mass of $\omega\pi\pi$ is shown in Fig.1.

It is the same as the decay mode studied above, only the axial-vector currents contribute

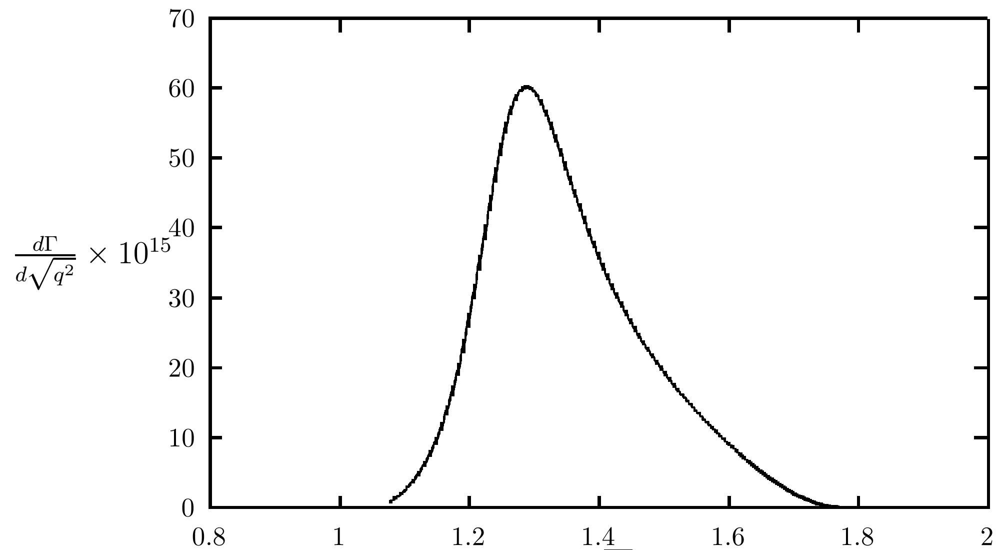

FiG.1 $\sqrt{q^2}$ GeV

to $\tau \to \omega\rho\nu$. At the tree level $\mathcal{L}^{\omega\rho\pi}$ (20)is the only vertex involved in this decay channel. Therefore, the decay is resulted by the WZW anomaly. Using the term

$$-\frac{g_W}{4} cos\theta_C f_\pi A^i_\mu \partial^\mu \pi^i$$

in Eq.(3) and the vertex(20), the matrix element of the axial-vector current is obtained

$$< \omega\rho^- | \bar{\psi}\tau_+\gamma_\mu\gamma_5\psi | 0 >= -\frac{i}{\sqrt{4E_1E_2}} \frac{N_C}{\pi^2 g^2} \frac{q_\mu}{q^2} \varepsilon^{\lambda\nu\alpha\beta} p_{1\lambda} p_{2\nu} \epsilon^*_\alpha(p_1) \epsilon^*_\beta(p_2). \tag{23}$$

In the limit $m_q = 0$, the axial-vector current is not conserved in this process. The conservation of the matrix element(22) is resulted by the cancellations between the terms $\frac{1}{f_a} j^{iW}_\mu$, $\Delta m^2 f_a a^i_\mu$, and $f_\pi \partial_\mu \pi^i$ of $\mathcal{L}^A(5)$(see Eq.(34) of Ref.[6]). In the matrix element(13) there is

cancellation between the two vertices(11), however, the cancellation is not enough. This result leads to the nonconservation of the axial-vector current in Eq.(13). For the process(23) there is no contribution from a_1 meson, therefore, there is no such cancellation. The pion exchange dominates this decay and the axial-vector current is not conserved. The divergence of the operator used to derive the matrix element(23) is

$$\frac{N_C}{\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\rho^i_\beta \tag{24}$$

Using the matrix element (23) and adding the Breit-Wigner formula of the ρ meson in, the decay width is obtained

$$\Gamma = \frac{G^2}{128m_\tau}\frac{cos^2\theta_C}{(2\pi)^3}\frac{9}{\pi^4 g^4}\int_{q^2_{min}}^{m^2_\tau} dq^2\frac{1}{q^6}(m^2_\tau - q^2)^2$$
$$\int_{4m^2_\pi}^{(\sqrt{q^2}-m_\omega)^2} dk^2[(q^2+m^2_\omega-k^2)^2-4q^2m^2_\omega]^{\frac{3}{2}}\frac{1}{\pi}\frac{\sqrt{k^2}\Gamma_\rho(k^2)}{(k^2-m^2_\rho)^2+k^2\Gamma^2_\rho(k^2)}, \tag{25}$$

where $q^2_{min} = 2m_\pi m_\tau + \frac{m_\tau m^2_\omega}{m_\tau - 2m_\pi}$, k^2 is the invariant mass of the two pions and

$$\Gamma_\rho(k^2) = \frac{f^2_{\rho\pi\pi}(k^2)}{48\pi}\frac{k^2}{m_\rho}(1-4\frac{m^2_\pi}{k^2})^3,$$
$$f_{\rho\pi\pi}(k^2) = \frac{2}{g}\{1+\frac{k^2}{2\pi^2 f^2_\pi}[(1-\frac{2c}{g})^2-4\pi^2c^2]\}. \tag{26}$$

The branching ratio is computed to be

$$B = 0.16\times 10^{-4}.$$

The decay rate is much smaller than the one of $\tau\rightarrow\omega(\pi\pi)_{non\rho}\nu$. Therefore, $\tau\rightarrow\omega(\pi\pi)_{non\rho}\nu$ dominates the decay $\tau\rightarrow\omega\pi\pi\nu$. The distribution is shown in Fig.2.

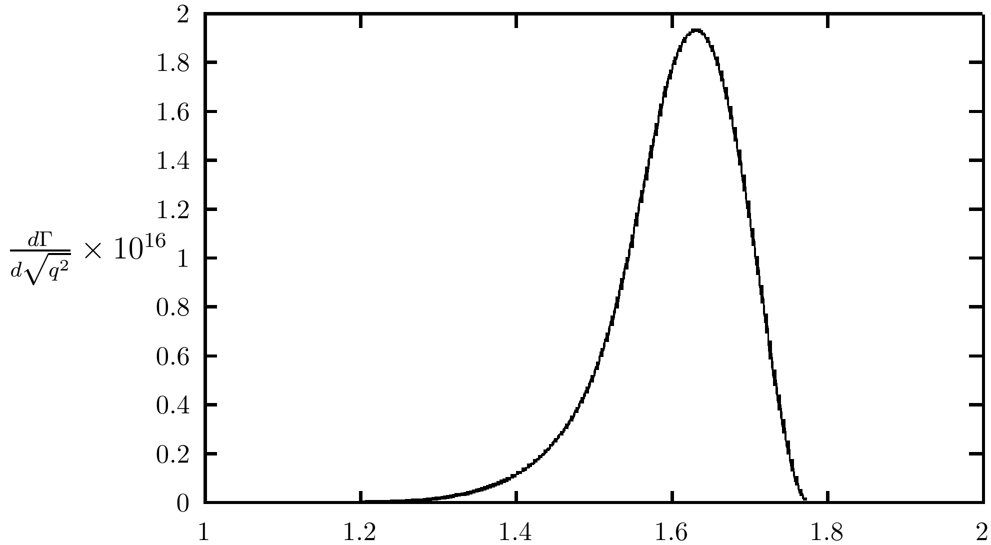

FiG.2 GeV

The nonconservation of the quark axial-vector currents found in the matrix element(13,23) show that there is anomaly in the PCAC. The Adler-Bell-Jackiw anomaly

$$\partial^\mu \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = \frac{\alpha}{4\pi}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\delta_{3i} \tag{27}$$

is well known. However, this anomaly is the one caused by electromagnetic interaction. The abnormal terms(14,24) are from strong interaction. We claim the existence of the strong anomaly in PCAC. Taking the abnormal term(23) as an example of the strong anomaly of the PCAC, the PCAC with strong anomaly is written as

$$\partial^\mu \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = -m_\pi^2 f_\pi \pi_i + \frac{N_C}{\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\rho^i_\beta. \tag{28}$$

The abnormal term of Eq.(28) is caused by the vertex(20) which originates in the WZW

anomaly. Why this vertex causes the anomaly of the PCAC? The reason is presented as below. As pointed in Ref.[6], the conservation of the quark axial-vector currents(in the chiral limit) is obtained from the cancellations between pion exchange, a_1 exchange, and others. In the Lagrangian of the WZW anomaly[4,5,7] there is no coupling like

$$\sim \varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\rho^i_\alpha a^i_\beta.$$

Therefore, the pion exchange provided by the vertex(20) cannot be canceled out. The lack of the cancellation leads to the nonconservation of the quark axial-vector currents in the chiral limit.

We use the effective chiral theory[5] to illustrate the existence of the strong anomaly of PCAC(28). The problem is similar to the one treated by Adler-Bell-Jackiw[1]. First it is necessary to show that in the chiral limit, if just applying the equations of motion to the quark axial-vector current, it is found that the currents are conserved in the chiral limit. The equations of motion are derived from the Lagrangian(6)(to avoid the $U(1)$ problem taking off the η field in this part of the discussion)

$$\partial^\mu\bar{\psi}\gamma_\mu = -i\bar{\psi}(\gamma_\mu v^\mu + \gamma_\mu\gamma_5 a^\mu - mu - M),$$

$$\gamma_\mu\partial^\mu\psi = i(\gamma_\mu v^\mu + \gamma_\mu\gamma_5 a^\mu - mu - M)\psi,$$

$$\rho^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\tau_i\gamma_\mu\psi \quad a^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi,$$

$$\Pi_i = i\sigma \frac{\bar{\psi}\tau_i\gamma_5\psi}{\bar{\psi}\psi}, \tag{29}$$

where $u = \sigma + i\gamma_5\tau\cdot\Pi$, $\sigma^2 = 1 - \Pi^2$. Using all these equations(29), to the leading order of the quark mass it is proved

$$\partial^\mu \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = -m_\pi^2 f_\pi \pi^i. \tag{30}$$

According to the Adler-Bell-Jackiw[1] the anomaly does not come from the classic equation of motion, instead, from renormalization of the quark triangle diagrams. Adding the photon field to the Lagrangian(6), the vector part of the Lagrangian is written as

$$\mathcal{L} = \bar{\psi}(x)\{i\gamma\cdot\partial + \gamma^\mu(\frac{1}{g}\tau_3\rho^0_\mu + \frac{1}{2}e\tau_3 A_\mu + \frac{1}{g}\omega_\mu + \frac{1}{6}eA_\mu)\}\psi(x) \tag{31}$$

This part of the Lagrangian shows that the ρ, ω and the photon fields are in symmetric positions which lead to the VMD[5]. As pointed out by Sakurai[8], the substitutions

$$\rho^i \to \frac{1}{2}egA, \quad \omega \to \frac{1}{6}egA \tag{32}$$

revealed from Eq.(31) are essential to obtain VMD. Taking ρ and ω fields as external fields and calculating the quark triangle diagrams as done by Adler-Bell-Jackiw, the strong anomaly of PCAC is derived as the one shown in Eq.(28). It is equivalent to say that this anomaly(28) can be found by using the substitution

$$Tr\tau^3 Q^2 \partial_\mu A_\nu \partial_\alpha A_\beta \to 2Tr\tau_3\tau_3\partial_\mu\omega_\nu\partial_\alpha\rho^3_\beta$$

in Eq.(27).

The abnormal term of Eq.(28) can be written as the divergence of the current

$$\frac{N_C}{2\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\{\omega_\nu\partial_\alpha\rho^i_\beta+\rho^i_\nu\partial_\alpha\omega_\beta\}. \tag{33}$$

Therefore, a question is raised that whether this current is part of the quark axial-vector current. If so, there is no strong interaction anomaly in PCAC. A direct proof is necessary. The method using the quark operator to bosonize the quark currents is presented in Ref.[5]. The couplings between ω meson and others obtained by using this method is the same as the one derived from the WZW Lagrangian[4,7]. Following this method, we have

$$<\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi>=-\frac{iN_C}{(2\pi)^D}\int d^Dp Tr\tau_i\gamma_\mu\gamma_5 s_F(x,p), \tag{34}$$

$$s_F(x,p)=s^0_F(p)\sum_{n=0}^{\infty}(-i)^n\{\gamma^\mu D_\mu s^0_F(p)\}^n, \tag{35}$$

$$D_\mu=\partial_\mu-iv_\mu-ia_\mu\gamma_5, \tag{36}$$

$$s^0_F(p)=-\frac{\gamma^\mu p_\mu-m\hat{u}}{p^2-m^2}, \tag{37}$$

$$\hat{u}=exp^{-i\gamma_5(\tau^i\pi^i+\eta)}. \tag{38}$$

The effective chiral theory of mesons[5] is a theory of mesons at low energies and the derivative expansion has been exploited. We are only interested in the terms associated with the

antisymmetric tensor $\varepsilon^{\mu\nu\alpha\beta}$. The leading terms are from $n = 3$

$$< \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi >= \frac{N_C}{(2\pi)^D}\int d^Dp\frac{1}{(p^2-m^2)^4}Tr\tau_i\gamma_\mu\gamma_5(\gamma\cdot p-m)\gamma\cdot D(\gamma\cdot p-m)\gamma\cdot D(\gamma\cdot p-m)\gamma\cdot D(\gamma\cdot p-m). \tag{39}$$

We are looking for the terms containing one ω and one ρ field only. The derivation shows that all nonzero terms are cancelled out. Therefore, the term(33) is not included in the quark axial-vector current. This conclusion is consistent with the fact that in the WZW Lagrangian there is no coupling between ω, ρ, and a_1 fields. The explanation is following. In the effective chiral theory of mesons[5] the couplings between a_1 fields and others are obtained from part of the Lagrangian(6)

$$a^i_\mu < \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi > .$$

As obtained above, in the effective axial-vector currents(39) there is no terms containing both ω and ρ fields only. Therefore, there is no $a_1\omega\rho$ coupling.

We emphasize on that the PCAC with strong anomaly(28) is model independent. The vertex(20) is derived from the WZW Lagrangian[4,5,7] and is very general. The term $-\frac{g_W}{4}cos\theta_C f_\pi A^i_\mu\partial^\mu\pi^i$ used to derived the matrix element (23) is independent of any model.

The vertex(20) has been well tested. In Ref.[5] it has been used to derive the amplitude of $\pi^0 \to 2\gamma$ obtained by Adler-Bell -Jackiw triangle anomaly. The decay rates of $\omega \to \pi\gamma$ and $\rho \to \pi\gamma$ are via VMD calculated by using this vertex and theoretical results are in good

agreements with data. It is also shown in Ref.[5] that this vertex is responsible for the decay $\omega 3\pi$.

On the other hand, using the substitutions(32) in Eq.(28) we derive

$$\partial^\mu \bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = -m_\pi^2 f_\pi \pi_i + \frac{N_C}{\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\rho_\beta^i + \frac{\alpha}{4\pi}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\delta_{3i}$$
$$+\frac{e}{4\pi^2 g}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\partial_\alpha\rho_\beta^i + \frac{3e}{4\pi^2 g}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu F_{\alpha\beta}\delta_{3i}. \tag{40}$$

It is well known that the amplitude of $\pi^0 \to 2\gamma$ is derived from the third term of the Eq.(41) under a soft pion approximation. In the same way, the amplitudes of $\omega \to \pi\gamma$ and $\rho \to \pi\gamma$ are obtained from the fourth and the fifth term of Eq.(40)

$$\mathcal{M}(\omega \to \pi\gamma) = \frac{3e}{2\pi^2 g}\varepsilon^{\mu\nu\alpha\beta}p_\mu k_\alpha \epsilon_\nu(p)\epsilon_\beta^*(k), \tag{41}$$

where k and p are momentum of ω and photon respectively.

$$\mathcal{M}(\rho \to \pi\gamma) = \frac{e}{2\pi^2 g}\varepsilon^{\mu\nu\alpha\beta}p_\mu k_\alpha \epsilon_\nu(p)\epsilon_\beta^*(k), \tag{42}$$

The decay widths are

$$\Gamma(\omega \to \pi\gamma) = \frac{3\alpha}{32\pi^4 g^2}\frac{m_\omega^3}{f_\pi^2}(1-\frac{m_\pi^2}{m_\omega^2})^3 = 583keV, \tag{43}$$
$$\Gamma(\rho \to \pi\gamma) = \frac{\alpha}{96\pi^4 g^2}\frac{m_\rho^3}{f_\pi^2}(1-\frac{m_\pi^2}{m_\rho^2})^3 = 61keV. \tag{44}$$

The data are $717(1 \pm 0.07)$keV and $67.8(1 \pm 0.12)$keV [10] respectively.

It is known that $\omega \to \rho\pi$ is dominant in the decay $\omega \to 3\pi$. In the manner of the calculations done above, the PCAC with anomaly(28) is used to calculate the decay rate of $\omega \to 3\pi$

$$\Gamma(\omega \to 3\pi) = \frac{1}{384m_\omega^3}\frac{1}{(2\pi)^3}\int dq_1^2 dq_2^2 (m_\omega^2 - q_1^2 - q_2^2)\{(m_\omega^2 - q_1^2)(m_\omega^2 - q_2^2)$$
$$-m_\omega^2(q_1^2 + q_2^2 - m_\omega^2)\}\{\frac{f_{\rho\pi\pi}^2(q_1^2)}{q_1^2 - m_\rho^2} + \frac{f_{\rho\pi\pi}^2(q_2^2)}{q_2^2 - m_\rho^2} + \frac{f_{\rho\pi\pi}^2(q_3^2)}{q_3^2 - m_\rho^2}\}^2. \tag{45}$$

In Eq.(45) the amplitude of $\omega \to 3\pi$ is determined in the chiral limit. The numerical result is

$$\Gamma(\omega \to 3\pi) = 7.7MeV.$$

The data is 7.49(1±0.02)MeV[10].

There are more terms on the right hand side of the Eq.(40). For example, the term(14) should be added to the right hand side of Eq.(28). In the same way obtaining the term(24), the vertex

$$\mathcal{L}^{f_1 a_1 \pi} = \frac{1}{\pi^2 f_\pi} f_a^2 \varepsilon^{\mu\nu\alpha\beta} f_\mu \partial_\nu \pi^i \partial_\alpha a_\beta^i \tag{46}$$

used to calculate the decay width of $\tau \to f\pi\nu$ in Ref.[6] results a term

$$-\frac{1}{\pi^2} f_a^2 \varepsilon^{\mu\nu\alpha\beta} \partial_\mu f_\nu \partial_\alpha a_\beta^i \tag{47}$$

which should be part of the strong anomaly of the PCAC. Now the PCAC with strong

anomaly takes the form

$$\partial^\mu\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi = -m_\pi^2 f_\pi\pi_i + \frac{N_C}{\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\rho^i_\beta$$
$$+\frac{6}{\pi^2 g f_\pi^2}\frac{2c}{g}(1-\frac{2c}{g})\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\partial_\nu\omega_\mu\partial_\alpha\pi^j\partial_\beta\pi^k - \frac{1}{\pi^2}f_a^2\varepsilon^{\mu\nu\alpha\beta}\partial_\mu f_\nu\partial_\alpha a^i_\beta. \quad (48)$$

Obviously, there are much more terms for the strong anomaly of the PCAC. The method deriving those abnormal terms is the same as the one used to obtained (14) and (24). By taking away the factor $-\frac{g_W}{4}cos\theta_C A^i_\mu$ from $\mathcal{L}^A(3)$, the axial-vector currents are obtained. Combining these currents with proper vertices of mesons, the nonconservetive currents, if they exist, could be found.

It is necessary to emphasize that the decay $\tau\rightarrow\omega\rho\nu$ provides a direct evidence of the strong anomaly of the PCAC. Therefore, the measurements of the decay rate and the distribution of the decay rate versus the invariant mass of $\omega\rho$ will evidence the existence of the strong anomaly in the PCAC.

The discussion of the strong anomaly of the PCAC in two flavor case can be extended to three flavors. The decay $\tau\rightarrow K^*\rho\nu$ and $K^*\omega\nu$ are related to the anomaly of the PCAC of $\Delta s = 1$. Both the vector and axial-vector currents contribute to these decays. The vertices contributing to the matrix elements of the vector currents come from the term

$$-\frac{1}{8}Trv_{\mu\nu}v^{\mu\nu} \quad (49)$$

of the effective Lagrangian of mesons obtained from the Lagrangian(6) in Ref.[5], where

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - \frac{i}{g}[v_\mu, v_\nu] - \frac{i}{g}[a_\mu, a_\nu],$$

$$v_\mu = \tau^i \rho^i_\mu + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + \lambda_a K^a_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu,$$

$$a_\mu = \tau^i a^i_\mu + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + \lambda_a K^a_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{1s\mu},$$

where $a = 4, 5, 6, 7$. There are additional normalization factors for ϕ, a_1, f, and f_{1s}, which can be found in Ref.[5]. The vertices $\mathcal{L}^{K^*\bar{K}^*v}$ is derived from Eq.(49)

$$\begin{aligned}
\mathcal{L}^{K^*\bar{K}^*v} = & \frac{\sqrt{3}i}{g}\{(\partial_\mu v^8_\nu - \partial_\nu v^8_\mu)(K^{-\mu}K^{+\nu} + \bar{K}^{0\mu}K^{0\nu}) + \\
& v^{8\nu}[(\partial_\mu K^-_\nu - \partial_\nu K^-_\mu)K^{+\mu} - (\partial_\mu K^+_\nu - \partial_\nu K^+_\mu)K^{-\mu} + (\partial_\mu \bar{K}^0_\nu - \partial_\nu \bar{K}^0_\mu)K^{0\mu} - (\partial_\mu K^0_\nu - \partial_\nu K^0_\mu)\bar{K}^{0\mu}]\} \\
& + \frac{i}{g}\{\sqrt{2}(\partial_\mu \rho^+_\nu - \partial_\nu \rho^+_\mu)K^{-\mu}K^{0\nu} - \sqrt{2}(\partial_\mu \rho^-_\nu - \partial_\nu \rho^-_\mu)K^{+\mu}\bar{K}^{0\nu} + (\partial_\mu \rho^0_\nu - \partial_\nu \rho^0_\mu)(K^{-\mu}K^{+\nu} + \bar{K}^{0\mu}K^{0\nu}) \\
& + \sqrt{2}\rho^{+\nu}[(\partial_\mu K^-_\nu - \partial_\nu K^-_\mu)K^{0\mu} - (\partial_\mu K^0_\nu - \partial_\nu K^0_\mu)K^{-\mu}] \\
& - \sqrt{2}\rho^{-\nu}[(\partial_\mu K^+_\nu - \partial_\nu K^+_\mu)\bar{K}^{0\mu} - (\partial_\mu \bar{K}^0_\nu - \partial_\nu \bar{K}^0_\mu)K^{+\mu}] \\
& + \rho^{0\nu}[(\partial_\mu K^0_\nu - \partial_\nu K^0_\mu)\bar{K}^{+\mu} - (\partial_\mu K^+_\nu - \partial_\nu K^+_\mu)K^{-\mu} \\
& - (\partial_\mu \bar{K}^0_\nu - \partial_\nu \bar{K}^0_\mu)K^{0\mu} + (\partial_\mu K^0_\nu - \partial_\nu K^0_\mu)\bar{K}^{-\mu}]\},
\end{aligned} \tag{50}$$

where

$$v^8 = \frac{1}{\sqrt{3}}\omega - \frac{2}{\sqrt{3}}\phi.$$

Using the substitutions[5]

$$\rho^0 \to \frac{1}{2}egA, \quad \omega \to \frac{1}{6}egA, \quad \phi \to -\frac{1}{3\sqrt{2}}egA, \tag{51}$$

it is proved that the charges of K^{*+-0} are $+1$, -1, and 0 respectively, where A is the photon field. It is interesting to notice that the vertices(50) are from the nonabelian nature of the vector meson fields. The vertices $\mathcal{L}^{K^*\bar{K}^*\rho}$ and $\mathcal{L}^{K^*\bar{K}^*\omega}$ are found from Eq.(50). Using $\mathcal{L}^V(1)$, it is obtained

$$< \rho^0 K^{*-}|\bar{\psi}\lambda_+\gamma_\mu\psi|0 >= \frac{1}{2\sqrt{4E_1E_2}} sin\theta_C \frac{-m_{K^*}^2 + i\sqrt{q^2}\Gamma_{K^*}(q^2)}{q^2 - m_{K^*}^2 + i\sqrt{q^2}\Gamma_{K^*}q^2)}$$
$$(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})\{(k-p)^\nu \epsilon_\sigma(k)\epsilon^\sigma(p) + 2p^\sigma\epsilon^\nu(p)\epsilon_\sigma(k) - 2k^\sigma\epsilon^\nu(k)\epsilon_\sigma(p)\}, \tag{52}$$

where k and p are the momentum of ρ and K^* respectively, $q = k + p$,

$$\Gamma_{K^*}(q^2) = \frac{f_{\rho\pi\pi}^2(q^2)}{8\pi\sqrt{q^2}m_{K^*}}\{\frac{1}{4q^2}(q^2 + m_K^2 + m_\pi^2)^2 - m_K^2\}^{\frac{3}{2}}. \tag{53}$$

The vertices contributing to the matrix element of the quark axial-vector currents are from the WZW anomaly[5]

$$\mathcal{L}^{K^*Kv} = -\frac{N_C}{\pi^2 g^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta} d_{abc} K_\mu^a \partial_\nu v_\alpha^c \partial_\beta K^b - \frac{N_C}{\pi^2 g^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta} K_\mu^a \partial_\nu v_\alpha \partial_\beta K^a, \tag{54}$$

where v_α is the singlet. The vertices $K^*K\rho$ and $K^*K\omega$ are found from Eq.(54). Using these vertices and $\mathcal{L}^A(3)$, the matrix element of the axial-vector currents is obtained

$$< K^{*-}\rho^0|\bar{\psi}\lambda_+\gamma_\mu\gamma_5\psi|0 >= sin\theta_C < K^{*-}\rho^0|f_K\partial_\mu K^+|0 >=$$
$$\frac{i}{\sqrt{4E_1E_2}} sin\theta_C \frac{N_C}{2\pi^2 g^2}\frac{q_\mu}{q^2}\varepsilon^{\lambda\nu\alpha\beta} k_\nu \epsilon_\alpha(k)\epsilon_\lambda(p) q_\beta. \tag{55}$$

In the chiral limit, $f_K = f_\pi$. Obviously, the axial-vector current is not conserved in the limit of $m_q = 0$. The divergence of the term contributing to this matrix element is written as

$$-\frac{N_C}{2\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu K^{*-}_\nu \partial_\alpha \rho^0_\beta. \tag{56}$$

Adding the Breit-Wigner distribution of the ρ meson and using the matrix elements(52,55), the distributions of the decay $\tau \to K^{*-}\rho^0\nu$ versus the invariant mass of $K^*\rho$ are obtained

$$\begin{aligned}
\frac{d\Gamma^V}{dq^2} &= \frac{G^2}{(2\pi)^3}\frac{sin^2_{\theta_C}}{32m^3_\tau q^4}(m^2_\tau - q^2)^2 \int^{(\sqrt{q^2}-m_{K^*})^2}_{4m^2_\pi} dk^2 \{\frac{1}{4}(q^2+k^2-m^2_{K^*})^2 - q^2k^2\}^{\frac{3}{2}} \frac{m^2_\tau + 2q^2}{q^2k^2m^2_{K^*}} \\
&\frac{1}{\pi}\frac{\sqrt{k^2}\Gamma_\rho(k^2)}{(k^2-m^2_\rho)^2 + k^2\Gamma^2_\rho(k^2)}\frac{m^4_{K^*} + q^2\Gamma^2_{K^*}(q^2)}{(q^2-m^2_{K^*})^2 + q^2\Gamma^2_{K^*}(q^2)} \\
&\{q^2(k^2+m^2_{K^*}) + k^2m^2_{K^*} + \frac{1}{12}(q^2+k^2-m^2_{K^*})^2 - \frac{1}{3}q^2k^2\},
\end{aligned} \tag{57}$$

$$\begin{aligned}
\frac{d\Gamma^A}{dq^2} &= \frac{G^2}{(2\pi)^3}\frac{sin^2_{\theta_C}}{32m^3_\tau q^4}(m^2_\tau - q^2)^2 \int^{(\sqrt{q^2}-m_{K^*})^2}_{4m^2_\pi} dk^2 \{\frac{1}{4}(q^2+k^2-m^2_{K^*})^2 - q^2k^2\}^{\frac{3}{2}} \\
&\frac{2}{q^2}m^2_\tau(\frac{N_C}{2\pi^2 g^2})^2\frac{1}{\pi}\frac{\sqrt{k^2}\Gamma_\rho(k^2)}{(k^2-m^2_\rho)^2 + k^2\Gamma^2_\rho(k^2)},
\end{aligned} \tag{58}$$

where k^2 is the invariant mass of the two pions and $\Gamma^{V,A}$ are the decay widths from the vector and axial-vector currents respectively.

The branching ratio is computed to be 0.24×10^{-6}. The contribution of the axial-vector currents is 17%. The branching ratio of $\tau \to K^{*-}\omega\nu$ is determined to be 0.6×10^{-7} and the branching ratio of $\tau \to \bar{K}^{*0}\rho^-\nu$ is 0.48×10^{-6}.

The term(56) is the strong anomaly of the PCAC($\Delta s = 1$). From Eq.(50) other three

terms of the strong anomaly are found

$$-\frac{N_C}{2\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\{\sqrt{2}\partial_\mu \bar{K}^{*0}_\nu \partial_\alpha \rho^-_\beta + \partial_\mu K^{*-}_\nu \partial_\alpha \omega_\beta + \sqrt{2}\partial_\mu K^{*-}_\nu \partial_\alpha \phi_\beta\}. \tag{59}$$

Therefore, the PCAC($\Delta s = 1$) with strong anomaly is expressed as

$$\begin{aligned}\partial^\mu \bar{\psi}\lambda_+ \gamma_\mu \gamma_5 \psi &= -m_K^2 f_K K^- \\ &-\frac{N_C}{2\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\{\partial_\mu K^{*-}_\nu \partial_\alpha \rho^0_\beta + \sqrt{2}\partial_\mu \bar{K}^{*0}_\nu \partial_\alpha \rho^-_\beta + \partial_\mu K^{*-}_\nu \partial_\alpha \omega_\beta + \sqrt{2}\partial_\mu K^{*-}_\nu \partial_\alpha \phi_\beta\}. \end{aligned}\tag{60}$$

Using the VMD and the substitutions(51), the electromagnetic anomaly is added to the Eq.(60)

$$\begin{aligned}\partial^\mu \bar{\psi}\lambda_+ \gamma_\mu \gamma_5 \psi &= -m_K^2 f_K K^- - \frac{e}{2\pi^2 g}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu K^{*-}_\nu \partial_\alpha A_\beta \\ &-\frac{N_C}{2\pi^2 g^2}\varepsilon^{\mu\nu\alpha\beta}\{\partial_\mu K^{*-}_\nu \partial_\alpha \rho^0_\beta + \sqrt{2}\partial_\mu \bar{K}^{*0}_\nu \partial_\alpha \rho^-_\beta + \partial_\mu K^{*-}_\nu \partial_\alpha \omega_\beta + \sqrt{2}\partial_\mu K^{*-}_\nu \partial_\alpha \phi_\beta\}. \end{aligned}\tag{61}$$

In terms of the soft pion approximation the decay amplitude of $K^{*-} \to K^- \gamma$ is derived from Eq.(61) in the chiral limit

$$\mathcal{M} = -\frac{e}{2\pi^2 g f_\pi}\varepsilon^{\mu\nu\alpha\beta} p_\mu \epsilon_\nu(p) k_\alpha \epsilon^*_\beta(k). \tag{62}$$

The decay width is found to be

$$\Gamma = \frac{\alpha}{96\pi^4 g^2 f_\pi^2} m^3_{K^*}(1 - \frac{m_K^2}{m^2_{K^*}})^3 = 34.9keV. \tag{63}$$

The data[10] is 50.3(1±0.11)keV. The strange quark mass correction is responsible for the difference between the theoretical result and the experiment.

To conclude, the decay $\tau \to \omega\pi\pi\nu$ is resulted from the WZW anomaly. Theoretical result of the decay rate agrees with data. In the chiral limit, the strong anomaly of the PCAC is found and the strong anomaly originates in the WZW anomaly. Under the soft pion approximation the decay rates of $\omega \to \pi\gamma$, $\rho \to \pi\gamma$, and $\omega \to 3\pi$ obtained by using the PCAC with strong anomaly are in good agreement with data. The strong anomaly of the PCAC leads to the pion dominance in the decay $\tau \to \omega\rho\nu$. The measurements of the decay rate and distribution of $\tau \to \omega\rho\nu$ will provide a further evidence on the strong anomaly of the PCAC. The strong anomaly of the PCAC($\Delta s = 1$) exits too. It is necessary to emphasize that the Adler-Bell-Jackiw anomaly and the Adler-Bardeen anomaly of QCD are exact and the strong anomaly of the PCAC is based on that the meson fields are treated as effective point fields, like the pion fields in the PCAC and the meson fields in the WZW anomaly. However, the existence of the strong anomaly of the PCAC indicates that in the chiral limit, the quark octet axial-vector currents are not conserved. The expressions of the strong anomaly presented in this paper, maybe, are some kind of bosonization of the strong anomaly of the quark octet axial-vector currents.

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Appendix

$$\frac{d\Gamma(\tau \rightarrow \omega\pi\pi\nu)}{dq^2dk^2dk^{'2}} = \frac{G^2}{256m_\tau^3}\frac{cos^2\theta_C}{(2\pi)^5}\frac{1}{q^2}(m_\tau^2 - q^2)^2\{(m_\tau^2 + 2q^2)F + m_\tau^2 G\}, \tag{64}$$

where

$$F = \{\frac{2}{3}\frac{1}{q^4}(q\cdot k_1)^2(q^2 - q\cdot k_2)^2 + \frac{4}{9}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^2\}A_1$$
$$+\{\frac{2}{3}\frac{1}{q^4}(q\cdot k_2)^2(q^2 - q\cdot k_1)^2 + \frac{4}{9}\frac{1}{q^4}(q\cdot k_2)^2(q\cdot k_1)^2\}A_2$$
$$+\{\frac{2}{3}\frac{1}{q^2}q\cdot k_1 q\cdot k_2 q\cdot(k_1 + k_2) - \frac{10}{9}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^2\}A_{12} + \frac{2}{9}\frac{1}{q^4}(q\cdot k_1)^3(q\cdot k_2)^2 B_{11}$$
$$-\frac{2}{9}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^2 q\cdot(q - k_2)B_{21} + \frac{2}{9}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^2 q\cdot(q - k_1)B_{12}$$
$$-\frac{2}{9}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^3 B_{22} + \frac{2}{9}\frac{1}{q^4}\{(q\cdot k_1)^4(q\cdot k_2)^2 B_1 + (q\cdot k_1)^2(q\cdot k_2)^4 B_2\}BW_a$$
$$G = [\frac{12}{\pi^2 g^2 f_\pi^2}\frac{c}{g}(1 - \frac{c}{g})]^2\frac{8}{3}\frac{1}{q^4}(q\cdot k_1)^2(q\cdot k_2)^2$$
$$BW_a = \frac{9}{\pi^4 g^2 f_\pi^2}\frac{g^4 f_a^2 m_\rho^4 + f_a^{-2}q^2\Gamma_a^2(q^2)}{(q^2 - m_a^2)^2 + q^2\Gamma_a^2(q^2)},$$
$$BW_\rho(k^2) = \frac{1}{(k^2 - m_\rho^2)^2 + k^2\Gamma_\rho^2(k^2)},$$
$$A_1 = \{[\frac{2}{f_\pi}(1 - \frac{2c}{g}) + \frac{1}{gf_a}(k^{'2} - m_\rho^2)A(k^{'2})BW_\rho(k^{'2})]^2 + \frac{1}{g^2 f_a^2}A^2(k^{'2})k^{'2}\Gamma_\rho^2(k^{'2})BW_\rho(k^{'2})\},$$
$$A_2 = \{[\frac{2}{f_\pi}(1 - \frac{2c}{g}) + \frac{1}{gf_a}(k^2 - m_\rho^2)A(k^2)BW_\rho(k^2)]^2 + \frac{1}{g^2 f_a^2}A^2(k^2)k^2\Gamma_\rho^2(k^2)BW_\rho(k^2)\},$$
$$A_{12} = -2\{[\frac{2}{f_\pi}(1 - \frac{2c}{g}) + \frac{1}{gf_a}(k^{'2} - m_\rho^2)A(k^{'2})BW_\rho(k^{'2})][\frac{2}{f_\pi}(1 - \frac{2c}{g})$$
$$+\frac{1}{gf_a}(k^2 - m_\rho^2)A(k^2)BW_\rho(k^2)] + \frac{1}{g^2 f_a^2}A^2(k^{'2})A^2(k^2)\sqrt{k^2 k^{'2}}\Gamma_\rho(k^{'2})BW_\rho(k^2)\Gamma_\rho(k^{'2})BW_\rho(k^{'2})\},$$
$$B_{11} = 2B\{[\frac{2}{f_\pi}(1 - \frac{2c}{g}) + \frac{1}{gf_a}(k^{'2} - m_\rho^2)A(k^{'2})BW_\rho(k^{'2})]\frac{1}{gf_a}(k^2 - m_\rho^2)BW_\rho(k^2)]$$
$$+\frac{1}{g^2 f_a^2}A^2(k^{'2})\sqrt{k^2 k^{'2}}\Gamma_\rho(k^{'2})BW_\rho(k^2)\Gamma_\rho(k^{'2})BW_\rho(k^{'2})\},$$

$$
\begin{aligned}
B_{12} &= -2B\{[\frac{2}{f_\pi}(1-\frac{2c}{g})+\frac{1}{gf_a}(k^2-m_\rho^2)A(k^2)BW_\rho(k^2)]\frac{1}{gf_a}(k^2-m_\rho^2)BW_\rho(k^2)] \\
&+\frac{1}{g^2f_a^2}A(k^2)k^2\Gamma_\rho^2(k^2)BW_\rho^2(k^2)\} \\
B_{21} &= 2B\{[\frac{2}{f_\pi}(1-\frac{2c}{g})+\frac{1}{gf_a}(k^{'2}-m_\rho^2)A(k^{'2})BW_\rho(k^{'2})]\frac{1}{gf_a}(k^{'2}-m_\rho^2)BW_\rho(k^{'2})] \\
&+\frac{1}{g^2f_a^2}A(k^{'2})k^{'2}\Gamma_\rho^2(k^{'2})BW_\rho^2(k^{'2})\}, \\
B_{22} &= -2B\{[\frac{2}{f_\pi}(1-\frac{2c}{g})+\frac{1}{gf_a}(k^2-m_\rho^2)A(k^2)BW_\rho(k^2)]\frac{1}{gf_a}(k^{'2}-m_\rho^2)BW_\rho(k^{'2})] \\
&+\frac{1}{g^2f_a^2}A^2(k^2)\sqrt{k^2k^{'2}}\Gamma_\rho(k^{'2})BW_\rho(k^2)\Gamma_\rho(k^{'2})BW_\rho(k^{'2})\}, \\
B_1 &= B^2\frac{1}{g^2f_a^2}BW_\rho(k^2), \\
B_2 &= B^2\frac{1}{g^2f_a^2}BW_\rho(k^{'2}),
\end{aligned}
\tag{65}
$$

where $q=p+k_1+k_2$, $k^2=(q-k_1)^2$, $k^{'2}=(q-k_2)^2$.

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Fig.1 Caption

Distribution of the decay rate of $\tau \rightarrow \omega\pi\pi\nu$ vs. the invariant mass of $\omega\pi\pi$

Fig.2 Caption

Distribution of the decay rate of $\tau \rightarrow \omega\rho\nu$ vs. the invariant mass of $\omega\rho$